Feasibility conditions and a table

- divisibility conditions
- integrality of eigenvalues
- integrality of multiplicities
- Krein conditions
- Absolute bounds

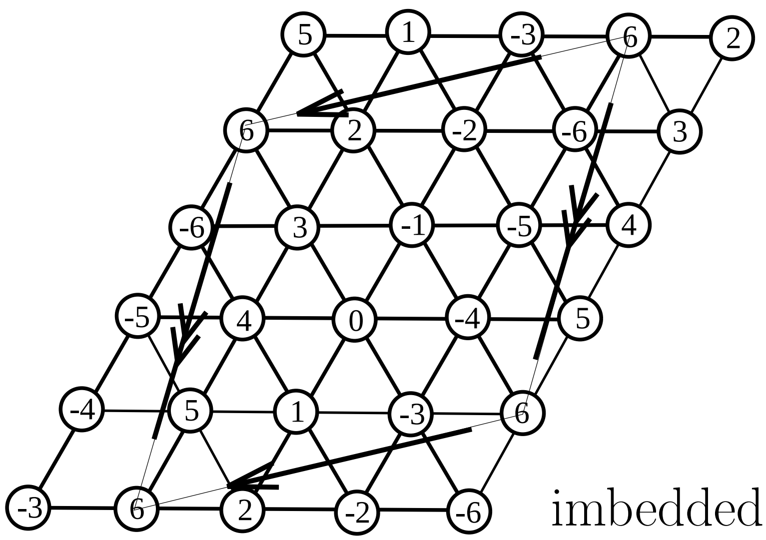
$$n \leq \frac{1}{2} m_\sigma (m_\sigma + 3),$$

and if $q_{11}^1 \neq 0$ even

$$n \leq \frac{1}{2} m_\sigma (m_\sigma + 1).$$

	n	k	λ	μ	σ	τ	m_σ	m_τ	graph
!	5	2	0	1	$\frac{-1+\sqrt{5}}{2}$	$\frac{-1-\sqrt{5}}{2}$	2	2	$C_5 = P(5)$ - Seidel
!	9	4	1	2	1	-2	4	4	$C_3 \times C_3 = P(9)$
!	10	3	0	1	1	-2	5	4	Petersen=compl. $T(5)$
!	13	6	2	3	$\frac{-1+\sqrt{13}}{2}$	$\frac{-1-\sqrt{13}}{2}$	6	6	$P(13)$
!	15	6	1	3	1	-3	9	5	GQ(2,2)=compl. $T(6)$
!	16	5	0	2	1	-3	10	5	Clebsch
2!	16	6	2	2	2	-2	6	9	Shrikhande, $K_4 \times K_4$
!	17	8	3	4	$\frac{-1+\sqrt{17}}{2}$	$\frac{-1-\sqrt{17}}{2}$	8	8	$P(17)$
!	21	10	3	6	1	-4	14	6	compl. T(7)
0	21	10	4	5	$\frac{-1+\sqrt{21}}{2}$	$\frac{-1-\sqrt{21}}{2}$	10	10	conference
!	25	8	3	2	3	-2	8	16	$K_5 \times K_5$
15!	25	12	5	6	2	-3	12	12	$P(25)$ (Paulus)
10!	26	10	3	4	2	-3	12	13	(Paulus)
!	27	10	1	5	1	-5	20	6	GQ(2,4)=compl. Schlaefli
4!	28	12	6	4	4	-2	7	20	$T(8)$ (Chang)
41!	29	14	6	7	$\frac{-1+\sqrt{29}}{2}$	$\frac{-1-\sqrt{29}}{2}$	14	14	$P(29)$, (Bussemaker & Spence)

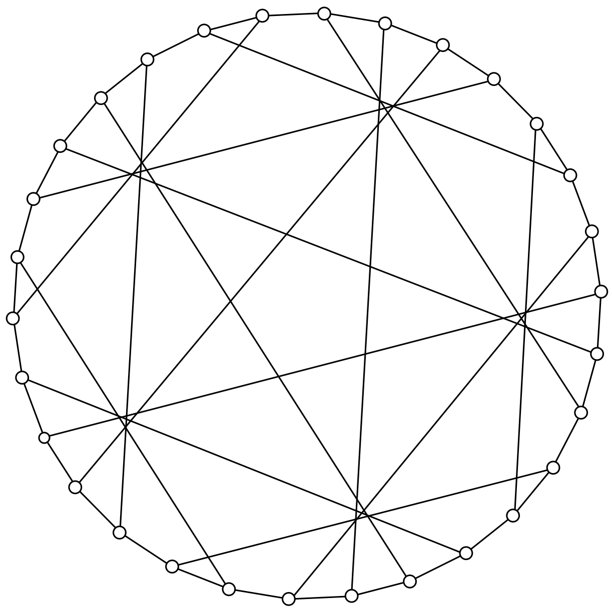
Paley graph $P(13)$



imbedded on a torus

The Shrikhande graph and $P(13)$ are the only distance-regular graphs which are locally C_6 (one has $\mu = 2$ and the other $\mu = 3$).

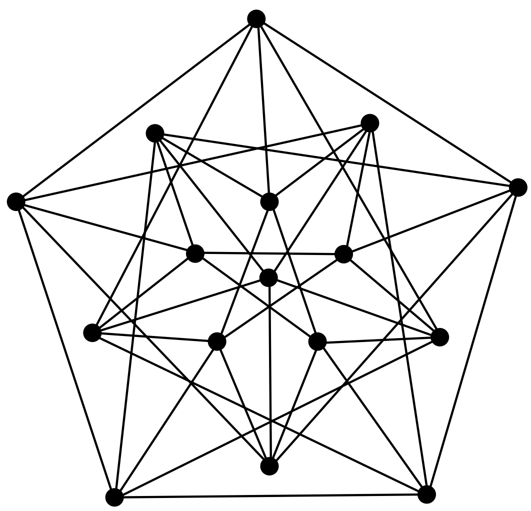
Tutte 8-cage

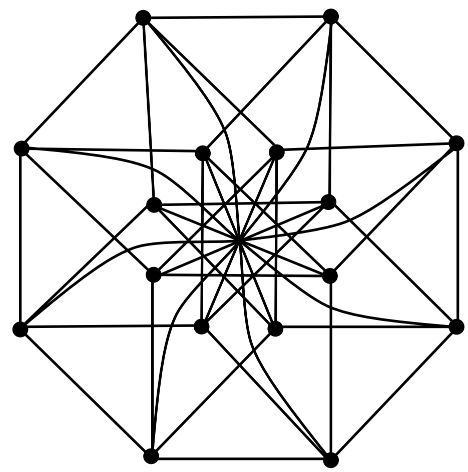


The Tutte's 8-cage is the $GQ(2,2) = W(2)$.

A **cage** is the smallest possible regular graph (here degree 3) that has a prescribed girth.

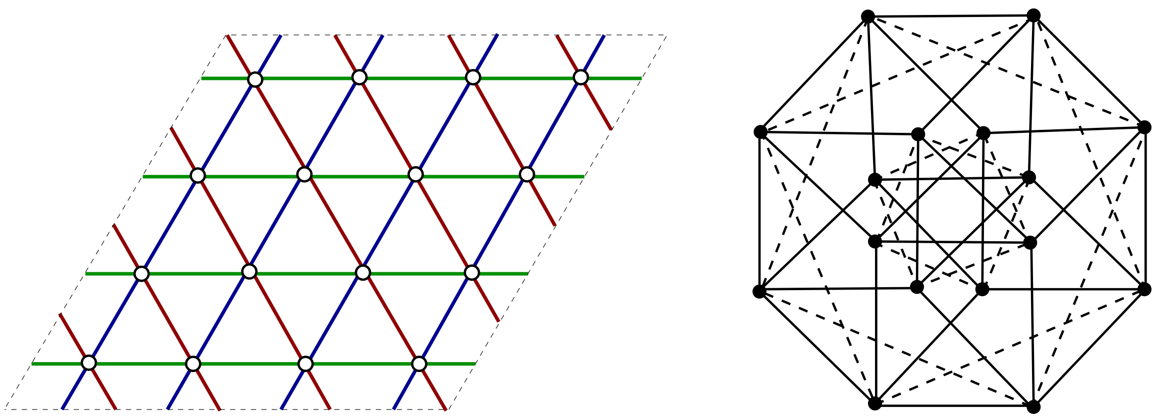
Clebsch graph





Two drawings of the complement of the Clebsch graph.

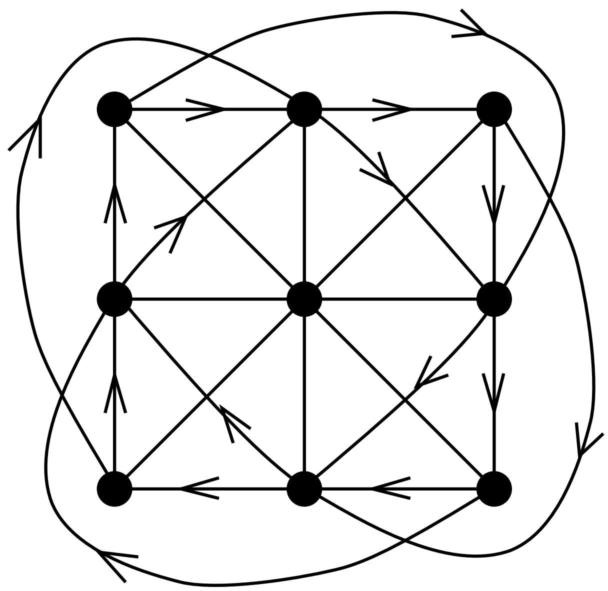
Shrikhande graph



The Shrikhande graph drawn on two ways:
(a) on a torus, (b) with imbedded four-cube.

The Shrikhande graph is not distance transitive, since some μ-graphs, i.e., the graphs induced by common neighbours of two vertices at distance two, are K_2 and some are $2\cdot K_1$.

Schläfly graph



How to construct the Schläfli graph:
make a cyclic 3-cover corresponding to arrows,
and then join vertices in every antipodal class.

Let Γ be a graph of diameter $\boldsymbol{d}$.

Then Γ has girth at most $\mathbf{2}\boldsymbol{d}+\mathbf{1}$,

while in the bipartite case the girth is at most $\mathbf{2}\boldsymbol{d}$.

Graphs with diameter d and girth $2d+1$ are called **Moore graphs** (**Hoffman and Singleton**).

Bipartite graphs with diameter d and girth $2d$ are known as **generalized polygons** (**Tits**).

A **Moore graph of diameter two** is a regular graph with girth five and diameter two.

The only Moore graphs are

- the pentagon,
- the Petersen graph,
- the Hoffman-Singleton graph, and
- possibly a strongly regular graph on 3250 vertices.